Optimal Midcourse Guidance for Medium-Range Air-to-Air Missiles

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An advanced midcourse guidance law for medium-range air-to-air missiles is proposed. The law consists of two different guidance modes: the final velocity maximum or the final time minimum, depending on the initial missile-target geometry. The former is preferable against a highly maneuverable target at a great distance, whereas the latter is demanded against a near target. This midcourse guidance law is combined with augmented proportional navigation in the homing phase. Performance is evaluated by computer simulations against conventional and advanced targets. The results show that the proposed guidance law is far superior to the guidance law that employs only augmented proportional navigation throughout the interception course, in regard to extending the launch boundaries or minimizing the interception time.

Nomenclature

C_D, C_L	= drag and lift coefficients, respectively
C_{D0}	= zero-lift drag coefficient
$C_{L\alpha}$	= $\partial C_L / \partial \alpha$
D	= drag
g	= acceleration of gravity
h, h_t	= missile and target altitudes, respectively
k	= induced drag coefficient
k_1, k_2	= penalty coefficients of performance index
L	= lift
M	= Mach number
m	= mass
r	= slant range between missile and target
s	= reference area
T	= thrust
t	= time
t_f	= interception time
t_{go}	= time-to-go
v	= velocity
x, x_t	= missile and target horizontal coordinates
α, α_0	= angle of attack and zero-lift angle
γ	= missile flight-path angle
ρ	= air density
ϕ	= performance index function
Ω	= stopping condition
$(\dot{\ })$	= time derivative

Subscripts

i, f	= initial and terminal values, respectively
max, min	= maximum and minimum values, respectively

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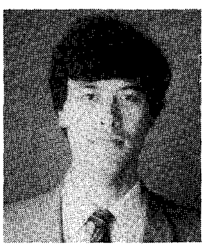

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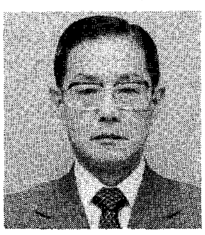

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Introduction

THE recent development of high-speed microprocessors has made it possible to apply optimal control theory to actual missile guidance systems, and many studies have appeared in this field. But only a few of them seem to have applied the concepts to midcourse guidance.[1,2] The purpose of this paper is to propose an optimal midcourse guidance law that employs different guidance modes depending on the required missile velocity and navigation time.

The main purpose of midcourse guidance is to navigate a missile so that it may operate in optimal conditions in regard to missile g performance and relative geometry against a target when seeker lock-on is achieved (the initial point of the homing phase). For the missile to achieve the necessary g performance, a certain minimum velocity is required, which differs corresponding to the target g performance, distance, and altitude. Against a target at a far distance or at a low altitude, missile velocity is a prime factor; therefore, the midcourse guidance law that maximizes the residual velocity is preferable. On the other hand, against a near target, the time margin is most important because the missile must destroy the target before it has a chance to attack its destination; therefore, the midcourse guidance law that minimizes the interception time is preferable. These guidance laws are obtained by solving nonlinear two-point boundary-value problems. A straightforward implementation of the law by an onboard computer is too complex, but once the necessary number of solutions is obtained by an off-line computer for several conditions and the obtained control is tabulated, then a real-time onboard implementation is possible by the interpolation of the nominal controls. The real-time correction is possible by solving the Riccati equation for the perturbed system. For example, the same technique introduced in Ref. 3 can be employed for this purpose.

In this paper, the mathematical model of a conceptual medium-range air-to-air missile is shown first. Second, the homing guidance laws are described. Third, the midcourse guidance laws are presented. After the required missile residual velocity is analyzed against a conventional and an advanced target, four types of midcourse guidance laws are explained, with their merits and demerits. Finally, various simulation results are shown using these guidance laws. The mixed strategy of the midcourse guidance laws, combined with augmented proportional navigation guidance (APNG)[4,5] in the homing phase, is compared with the other guidance law, which employs APNG throughout the interception course. The results are evaluated in relation to the interception time, the final missile velocity, and the launch boundaries.

Mathematical Model

Figure 1 shows missile force balance, as well as the symbols employed in this paper. For simplicity, motions are constrained within a vertical plane. The missile is modeled as a point mass, and the equations of motion are

$$\dot{v} = (T\cos\alpha - D)/m - g\sin\gamma \tag{1}$$

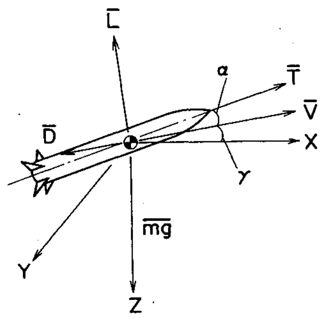

Fig. 1 Missile force balance and symbols.

$$\dot{\gamma} = (L + T\sin\alpha)/(mv) - g/v\cos\gamma \tag{2}$$

$$\dot{x} = v\cos\gamma \tag{3}$$

$$\dot{h} = v\sin\gamma \tag{4}$$

where

$$L = 1/2\,\rho v^2 s\, C_L, \qquad C_L = C_{L\alpha}\,(\alpha - \alpha_0) \tag{5}$$

$$D = 1/2\,\rho v^2 s\, C_D, \qquad C_D = C_{D0} + kC_L{}^2 \tag{6}$$

The aerodynamic derivative coefficients $C_{L\alpha}$, C_{D0}, and k are given as functions of Mach number M, which is a function of v and h,[6] and the air density ρ is a function of h.

$$\rho = \rho(h), \qquad M = M(v,h) \tag{7}$$

$$C_{L\alpha} = C_{L\alpha}(M), \qquad C_{D0} = C_{D0}(M), \qquad K = K(M) \tag{8}$$

The missile mass and thrust are given as functions of time t, as shown in Fig. 2,

$$m = m(t), \qquad T = T(t) \tag{9}$$

Homing Guidance

Before going into midcourse guidance, let us discuss the homing guidance law in the final course. Despite many discussions on the applicability of a linear optimal homing guidance law, it still is rather complex for real-time onboard implementation, whereas APNG seems to be promising for near-future application. As PNG is derived as a kind of optimal control against a nonmaneuvering target,[7] APNG[4] can be derived as a special case of optimal control against a maneuvering target.[5] The missile lateral acceleration command a_c of these guidance laws is given by[8]

PNG:

$$a_c = Ne\, v_c \dot{\sigma} \tag{10}$$

APNG:

$$a_c = Ne\,(v_c\dot{\sigma} + \tfrac{1}{2}a_t) \tag{11}$$

where Ne is the effective navigation ratio, v_c the closing velocity, a_t the lateral acceleration of the target, and $\dot{\sigma}$ the line-of-sight turning rate given by

$$v_c = -\dot{r} \tag{12}$$

$$\dot{\sigma} = (1/r^2)\,[(h_t - h_m)(\dot{x}_t - \dot{x}_m) - (\dot{h}_t - \dot{h}_m)(x_t - x_m)] \tag{13}$$

where r is the slant range

$$r = [(x_t - x_m)^2 + (h_t - h_m)^2]^{1/2} \tag{14}$$

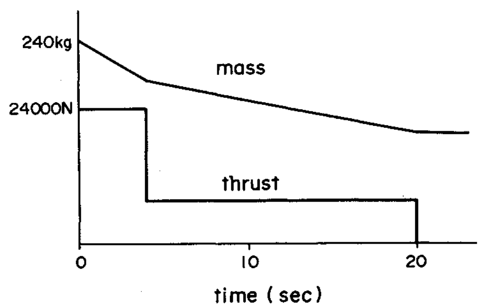

Fig. 2 Time histories of the missile mass and thrust.

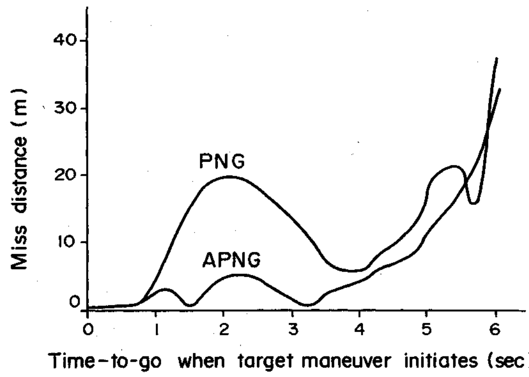

Fig. 3 Miss distance by a PNG and an APNG missile ($v_0 = 600$ m/s, $h = 3$ km, lock-on range = 5 km).

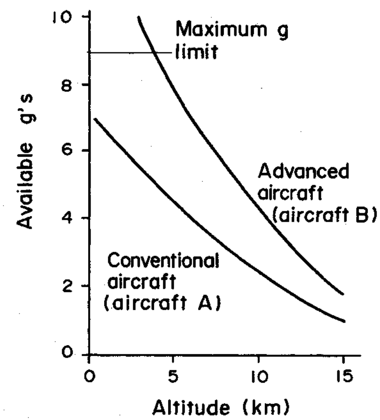

Fig. 4 The available lateral g of the aircraft.

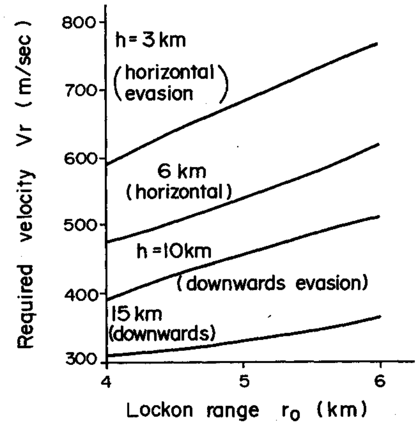

Fig. 5 Required missile velocity in relation to lock-on range and altitude (against aircraft A).

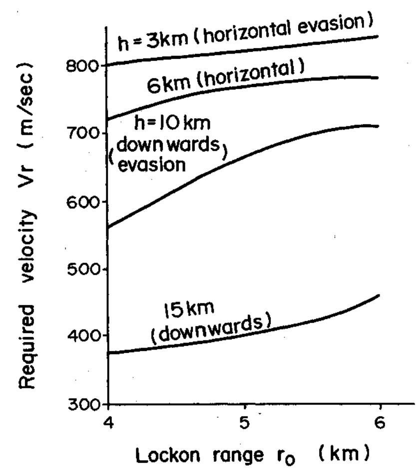

Fig. 6 Required missile velocity in relation to lock-on range and altitude (against aircraft B).

We see in Eq. (11) that APNG requires estimating the target lateral acceleration (usually by a Kalman filter). Once this estimation is obtained, the application to a current PNG missile is straightforward. Figure 3 shows a simulation example of the miss distance by a PNG and an APNG missile. The abscissa is the time-to-go when the target (a conventional aircraft, as will be explained in the next section) initiates the sustained maximum g turn. The initial relative range is 5 km; the initial missile and aircraft velocities are 600 m/s and 300 m/s, respectively; and the altitudes are both 3 km. In the PNG missile, the miss distance becomes large when an aircraft initiates a sustained maximum g turn (SMGT) 1 or 2 s before interception. The same tendency appears in the APNG missile, but the miss distance becomes fairly small. The reason is naturally thought to be the predicting term of the target acceleration in Eq. (11). This APNG in the homing phase is combined with the optimal midcourse guidance law in the following section and the performance is discussed.

Midcourse Guidance

A conventional aircraft (hereafter called aircraft A) and an advanced aircraft (aircraft B) are employed as target models that are different in maximum lateral g performance. Figure 4 shows the available lateral g of the assumed aircraft. Aircraft A has a performance of 7 g at sea level, whereas aircraft B has a performance of more than 9 g at an altitude of less than 3 km. The maximum g is limited to 9 g, considering pilot endurance.

To reduce the miss distance to less than, e.g., 10 m, a certain amount of residual velocity is required for a missile when the seeker lock-on is achieved. Study was conducted to calculate this velocity against a target that maneuvers with the lateral acceleration shown in Fig. 4 for various missile-target geometries. The results are summarized in Figs. 5 and 6. The abscissa shows the lock-on range when the missile starts the homing guidance, where the missile and the target are supposed to be at the same altitude. Generally, the missile must have a larger residual velocity against a target at a greater initial range or at a lower altitude.

Our previous study[9] showed that a downward maneuver, which makes the best use of gravity, is more advantageous than a horizontal or upward one. But the aircraft may not have enough time to pull up at low altitudes. Therefore, we assume here that the aircraft takes a horizontal SMGT at altitudes of less than 10 km, and a more advantageous downward SMGT (split-S) at altitudes of more than 10 km, where the aircraft can easily pull up again.

Figure 7 is another expression of Figs. 5 and 6, where the required missile minimum velocity vs altitude is shown against two aircraft types at the initial range of 5 km. Looking at the velocity at an altitude of 5 km, a missile needs 590 m/s v_r when the seeker lock-on is achieved against aircraft A, whereas 790 m/s v_r is required against aircraft B.

This minimum velocity affords just enough dynamic pressure to produce the necessary g for the missile. Figure 8 shows the relations between the required g performance of the employed missile at the angle of attack $\alpha = 10$ deg and altitude against two aircraft types.

Let us suppose that the interception point is predicted by the ground support system; then the missile can be navigated by the guidance laws that maximize the missile terminal velocity or minimize interception time. Mathematically, the optimal midcourse guidance laws are obtained by maximizing the following performance index ϕ, in relation to the missile angle of attack $\alpha(t)$, under the equations of missile motion (1-9)

$$\phi=(-k_1 t+k_2 v)_{t_f} \tag{15}$$

where the terminal time t_f is determined from the next stopping condition,

$$\Omega \equiv \tfrac{1}{2}[(x-x_t)^2+(h-h_t)^2-r_f^2]=0 \tag{16}$$

where r_f is the seeker lock-on range. By changing the k_1 and k_2 values in Eq. (15), we have the following two types of optimal control problems.

Final velocity maximum (v_f maximum):

$$k_1=0, \qquad k_2=1 \tag{17}$$

Final time minimum (t_f minimum):

$$k_1=1, \qquad k_2=0 \tag{18}$$

The problems are solved by the steepest ascent method.[10]

As for maximizing the final velocity, we found at least two types of optimal controls that produce local extremals. The first is the type in which the missile ascends sharply at first and then descends. The law usually produces the largest terminal velocity against a target at a far distance, but is time-consuming. Let us call this type I. The second is the type in which the missile ascends gradually and then falls down. This law also locally maximizes the missile terminal velocity, but less than with type I, and the navigation time is shorter. Let us call this type II. The optimal guidance law that minimizes the terminal time t_f is named type III. Although these optimal controls have their own merits, they are not suitable for homing guidance because the calculations of the missile control are time-consuming, even with the current high-speed digital computers. Therefore, we employ type I-type III guidance laws only in midcourse guidance: from the launch point through a near interception point where the missile seeker can lock onto the target. Following this, APNG is employed in the homing guidance. In comparison with the optimal guidance laws just cited, we will consider one that navigates the missile with the APNG throughout the interception course. We call this law type IV.

Now, the following four types of guidance laws are summarized.

Type I: Maximizing missile terminal velocity, sharp ascent and descent.

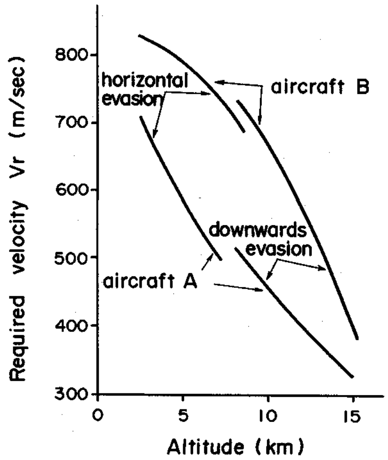

Fig. 7 Required missile minimum velocity vs altitude (lock-on range = 5 km).

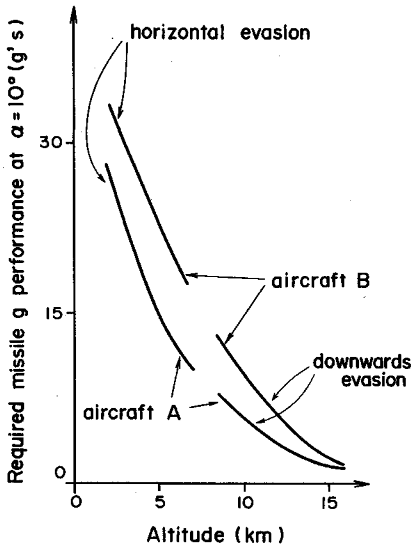

Fig. 8 Necessary missile g performance in relation to altitude (lock-on range = 5 km).

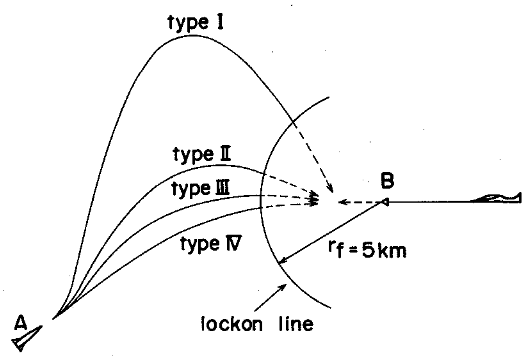

Fig. 9 The conceptual interception courses of the missiles under type I-IV guidance laws.

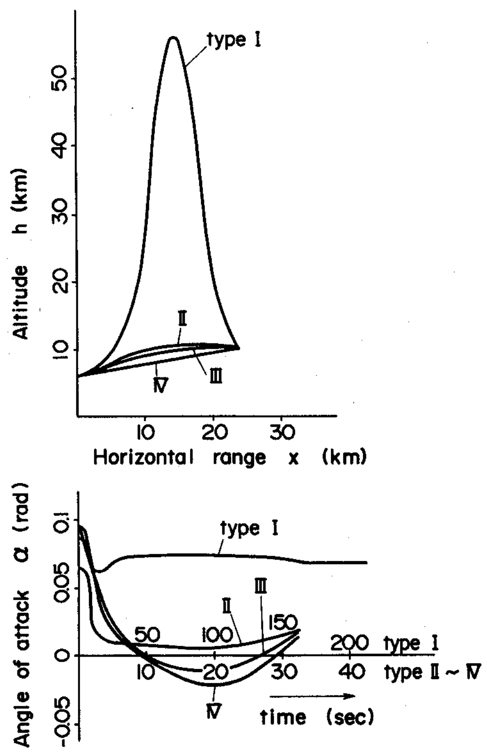

Fig. 10 The trajectories and control histories of the missile under type I-IV guidance laws (launch altitude = 6 km, x_f = 24 km, h_f = 10 km).

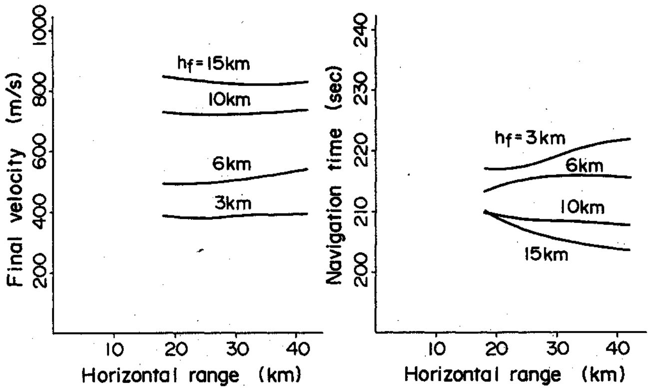

Fig. 11 The final velocity v_f and the navigation time t_f in relation to horizontal range x_f and final altitude h_f (launch altitude = 6 km, under type I guidance law).

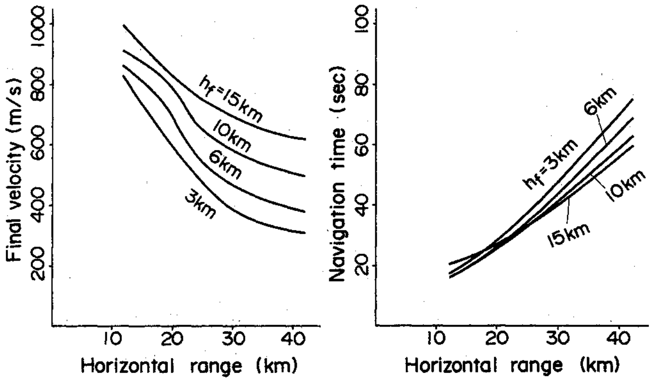

Fig. 13 The final velocity v_f and the navigation time t_f in relation to horizontal range x_f and final altitude h_f (launch altitude = 6 km, under type III guidance law).

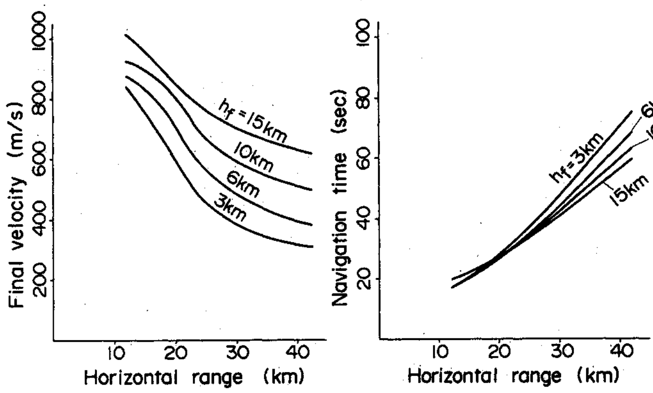

Fig. 12 The final velocity v_f and the navigation time t_f in relation to horizontal range x_f and final altitude h_f (launch altitude = 6 km, under type II guidance law).

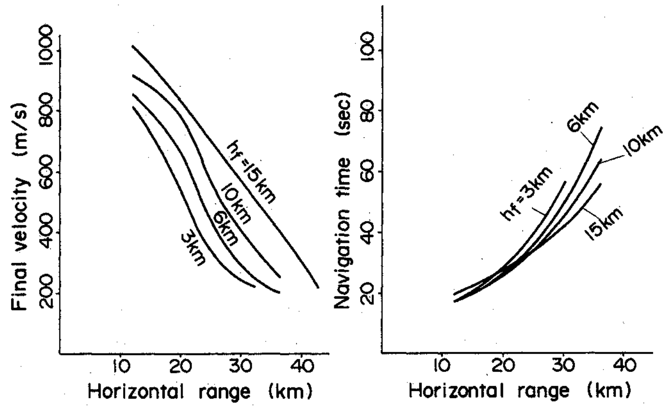

Fig. 14 The final velocity v_f and the navigation time t_f in relation to horizontal range x_f and final altitude h_f (launch altitude = 6 km, under type IV guidance law).

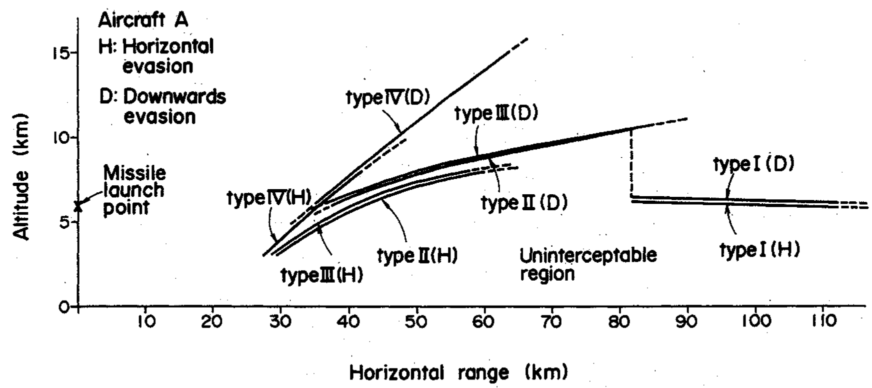

Fig. 15 The launch boundaries of the missile at 6-km altitude, under type I–IV guidance laws (against aircraft A).

Type II: Maximizing missile terminal velocity, gentle ascent and descent.

Type III: Minimizing terminal time.

Type IV: APNG (A representative of other advanced guidance laws.)

Figure 9 shows the conceptual interception courses with four guidance laws. Point A shows the initial launch point of the missile, and point B shows the target point when the missile enters the lock-on range. In the study, the lock-on range is assumed to be 5 km. Generally, the missile trajectories with type I control take a higher altitude than the other types.

Simulation Results and Discussions

The velocities and the navigation times at which the missile reaches the lock-on line are compared and evaluated.

Figure 10 shows an example of the typical trajectories and the control histories of the missile navigated by types I–IV guidance laws. The figure affirms the conceptual interception courses shown in Fig. 9 and gives numerical ideas on trajectories and angles of attack. Similar results are obtained for various missile launch altitudes.

The v_f and t_f values vs horizontal range attained with types I–IV guidance laws are shown in Figs. 11–14, respectively, where the missile is launched at 6-km altitude. The type I guidance law assures the largest v_f for larger values of the horizontal range, but the final time t_f becomes quite long. The type II guidance law produces a smaller v_f at larger ranges than type I but a larger value than types III and IV, where the t_f is not increased so much. Type III produces the minimum t_f. Type IV produces a comparable t_f but a smaller v_f compared with type II.

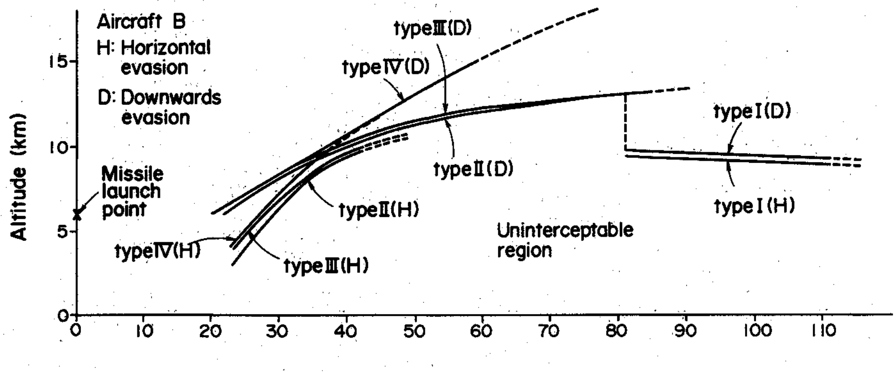

Fig. 16 The launch boundaries of the missile at 6-km altitude, under type I–IV guidance laws (against aircraft B).

From the results, we may say the following. When the target is detected at a far distance and there is sufficient time until interception, the type I guidance law should be employed as it produces the maximum v_f, and therefore has the highest g performance in the homing phase. On the other hand, when the target is detected at a near distance and small interception time is favorable, type III should be employed. Between these two situations, type II will be preferable as it allows the missile to intercept within the acceptable time and still assures a larger v_f than types III and IV. For a near target, type II brings a larger v_f than type I. In this case, type II naturally should be employed instead of type I.

We now propose the guidance law that employs an alternate strategy of types I–III at midcourse and APNG in the homing phase. To illustrate the performance of this guidance law, the missile launch boundaries are calculated by combining Figs. 11–13 with Fig. 7. By assuming an aircraft type and a lock-on altitude h_f, we know the required velocity v_r of the missile at lock-on from Fig. 7. Interpolating this v_r into Figs. 11–14, we obtain the horizontal range x_f and, again, by using this x_f in these figures, we obtain the navigation time t_f. As the aircraft is supposed to be closing with the velocity of 300 m/s, the approximate value of the horizontal range boundary of the aircraft position is calculated as $x_f + 0.3t_f + r_f$ (lock-on range).

Figures 15 and 16 show the launch boundaries of the missile at the initial altitude of 6 km, against aircraft A and B, under the proposed and the type IV guidance law. The area under or at the right side of boundaries shows the target position where the missile is unable to destroy the target because of lack of missile velocity even if the missile seeker can lock onto the target. The launch boundaries are more restricted against aircraft B than against aircraft A. But the proposed guidance law considerably extends the region beyond that of type IV. In the cases of Figs. 15 and 16, we choose type III when the target horizontal range is less than 20 km and type I when it is more than 85 km. Between 20 and 85 km, we choose type II. As the difference between types II and III is small, for simplicity, we may omit type II and employ type III instead of the former. Against an aircraft in the uninterceptable region, we have to wait until the target comes to the upper or left side of the boundaries.

Conclusions

The missile guidance law that employs the alternate strategy of either maximizing the final velocity or minimizing the final time (navigation time) in the midcourse phase, as well as augmented proportional navigation guidance in the homing phase, is proposed.

As two types of local optimum solutions in maximizing midcourse final velocity are found, the proposed guidance law selects one of the three types of guidance laws (two for maximizing final velocity, one for minimizing navigation time), depending on the situation.

The example of the launch boundaries of the missile navigated by the proposed guidance law is illustrated in comparison with the guidance law that employs augmented proportional navigation guidance throughout the interception course against conventional and advanced aircraft. The launch boundaries are more restricted against advanced aircraft than against conventional ones. But the proposed guidance law considerably extends the region compared to that of the latter guidance law.

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